Preface

This book, written as a treatise on mathematical finance, has two parts: deterministic and stochastic models.

The first part of the book, managed by Ernesto Volpe di Prignano, aims to give a complete presentation of the concepts and models of classical and modern mathematical finance in a mainly deterministic environment. Theoretical aspects and economic, bank and firm applications are developed.

The most important models are presented in detail after the formalization of an axiomatic theory of preferences. This performs the definition of “interest” and the financial regimes, which are the basis of financial evaluation and control the models. They are applied by means of clarifying examples with the solutions often obtained by *Excel* spreadsheet.

Chapter 1 shows how the fundamental definitions of the classical financial theory come from the microeconomic theory of subjective preferences, which afterwards become objective on the basis of the market agreements. In addition, the concepts of interest such as the price of other people’s money availability, of financial supply and the indifference curve are introduced.

Chapter 2 develops a strict mathematical formalization on the financial laws of interest and discount, which come from the postulates defined in Chapter 1. The main properties, i.e. decomposability and uniformity in time, are shown.

Chapter 3 shows the most often used financial law in practice. The most important parametric elements, such as interest rates, intensities and their relations, are defined. Particular attention is given to the compound interest and discount laws

in different ways. They find wide application in all the pluriennial financial operations.

Chapter 4 gives the concept of discrete time financial operation as a set of financial supplies, of operation value, of fair operation, of retrospective and prospective reserve at a given time, of the usufruct and bare ownership. In addition, a detailed classification of the financial projects based on their features is given. The decision and choice methods among projects are deeply developed. In the appendix to this chapter, a short summary of simple numerical methods, particularly useful to find the project internal rate, is reported.

Chapter 5 discusses all versions of the annuity operations in detail, as a particular case of financial movement with the same sign. The annuity evaluations are given using the compound or linear regime.

Chapter 6 is devoted to management mathematical procedures of financial operations, such as loan amortizations in different usual cases, the funding, the returns and the redemption of the bonds. Many *Excel* examples are developed. the final section is devoted to bond evaluations depending on a given rate or on the other hand to the calculus of return rates on bond investments.

In Chapters 7 and 8, the financial theory is reconsidered assuming variable interest rates following a given term structure. Thus, Chapter 7 defines spot and forward structures and contracts, the implicit relations among the parameters and the transforming formulae as well. Such developments are carried out with parameters referred to real and integer times following the market custom. Chapter 8 discusses the methods developed in Chapters 5 and 6 using term structures.

Chapter 9 is devoted to definition and calculus of the main duration indexes with examples. In particular, the importance of the so-called “duration” is shown for the approximate calculus of the relative variation of the value depending on the rate. However, the most relevant “duration” application is given in the classical immunization theory, which is developed in detail, calculating the optimal time of realization and showing in great detail the Fisher-Weil and Redington theorems.

The second part of the book, managed by Jacques Janssen and Raimondo Manca, aims to give a modern and self-contained presentation of the main models used in so-called *stochastic finance* starting with the seminal development of Black, Scholes and Merton at the beginning of the 1970s. Thus, it provides the necessary follow up of our first part only dedicated to the deterministic financial models.

However, to help in assuring the self-containment of the book, the first four chapters of the second part provide a summary of the basic tools on probability and

stochastic processes, semi-Markov theory and Itô's calculus that the reader will need in order to understand our presentation.

Chapter 10 briefly presents the basic tools of probability and stochastic processes useful for finance using the concept of *trajectory* or *sample path* often representing the time evolution of asset values in stock exchanges.

Chapters 11 and 12 summarize the main aspects of Markov and semi-Markov processes useful for the following chapters and Chapter 13 gives a strong introduction to stochastic or Itô's calculus, being fundamental for building stochastic models in finance and their understanding.

With Chapter 14, we really enter into the field of stochastic finance with the full development of classical models for option theory including a presentation of the Black and Scholes results and also more recent models for exotic options.

Chapter 15 extends some of these results in a semi-Markov modeling as developed in Janssen and Manca (2007).

With Chapter 16, we present another type of problem in finance, related to interest rate stochastic models and their application to bond pricing. Classical models such as the Ornstein-Uhlenbeck-Vasicek, Cox-Ingersoll-Ross and Heath-Jarrow-Morton models are fully developed .

Chapter 17 presents a short but complete presentation of Markowitz theory in portfolio management and some other useful models.

Chapter 18 is one of the most important in relation to Basel II and Solvency II rules as it gives a full presentation of the value at risk, called VaR, methodology and its extensions with practical illustrations.

Chapter 19 concerns one of the most critical risks encountered by banks: credit or default risk problems. Classical models by Merton, Longstaff and Schwartz but also more recent ones such as homogenous and non-homogenous semi-Markov models are presented and used for building ratings and following the time evolution.

Finally, Chapter 20 is entirely devoted to the presentation of Markov and semi-Markov reward processes and their application in an important subject in finance, called stochastic annuity.

As this book is written as a treatise in mathematical finance, it is clear that it can be read in sections in a variety of sequences, depending on the main interest of the reader.

This book addresses a very large public as it includes undergraduate and graduate students in mathematical finance, in economics and business studies, actuaries, financial intermediaries, engineers but also researchers in universities and RD departments of banking, insurance and industry.

Readers who have mastered the material in this book will be able to manage the most important stochastic financial tools particularly useful in the application of the rules of governance in the spirit of Basel II for banks and financial intermediaries and Solvency II for insurance companies.

Many parts of this book have been taught by the three authors in several universities: Université Libre de Bruxelles, Vrije Universiteit Brussel, University of West Brittany (EURIA) (Brest), Télécom-Bretagne (Brest), Paris 1 (La Sorbonne) and Paris VI (ISUP) Universities, ENST-Bretagne, University of Strasbourg, Universities of Rome (La Sapienza), Napoli, Florence and Pescara.

Our common experience in the field of solving financial problems has been our main motivation in writing this treatise taking into account the remarks of colleagues, practitioners and students in our various lectures.

We hope that this work will be useful for all our potential readers to improve their method of dealing with financial problems, which always are fascinating.